# Lesson 8.1: Body Mass Index and other Health Numbers
# Theme: Health and Risk

| Main Math Topic | Main Quantitative Reasoning Context | Tools and Technology |
|---|---|---|
| Applied use of mathematical models | Body Mass Index | Calculators |

## Prerequisite Assumptions

Before beginning this lesson, students should be able to

- perform basic arithmetic operations including understanding the order of operations in a complex equation.
- make unit conversions, specifically from inches to meters and from pounds to kilograms.
- understand the role of variables in a mathematical model.
- read news articles with complex formulas.

## Specific Objectives

Students will understand that

- Mathematical models provide a method to estimate values that are difficult to measure directly.
- Mathematical models, while useful, are often imperfect.

Students will be able to

- Use a multivariate mathematical model to estimate a value.

## Explicit Connections

- Body Mass Index and Body Fat Percentage are two ways to measure a person's size and weight. They take into account multiple variables, such as height, age, or gender, to classify people in different weight ranges. The values can be used as measurements of health.
- Mathematical models are often not perfectly accurate ways to measure values, but they are useful.

## Notes to Self

One thing I want to do during this lesson …

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One thing I want to pay attention to in my students' thinking …

One connection or idea I want to remember …

**Suggested Timeline**

| Duration | Activity | Suggested Structure |
|---|---|---|
| 14 minutes | Questions 1 – 4 | Small groups, class discussion |
| 14 minutes | Questions 5 – 8 | Small groups, class discussion |
| 7 minutes | Question 9 | Small groups, class discussion |
| 5 minutes | Making Connections | Class discussion |

## Special Notes

Students are requested to bring in the height, weight, BMI, and, if possible, age of professional athletes. Students will use the BMI and age to estimate the athletes' body fat percentages.

Below are a number of male and female athletes that can be used by students who did not find values on their own.

Brett Favre (6'4", 222 pounds, 27 BMI)
Annika Sorenstam (5'6", 125 pounds, 20 BMI)
Michael Jordan (6'6", 195 pounds, 23 BMI)
Jackie Joyner-Kersee (5'10", 154 pounds, 22 BMI)
Mohamed Ali (6'3" 210 pounds, 26 BMI)
Martina Navratilova (5'8", 144 pounds, 22 BMI)
Tiger Woods (6'1", 185 pounds, 24 BMI)
Mia Hamm (5'5", 125 pounds, 21 BMI)
Hulk Hogan (6'7", 302 pounds, 34 BMI)
The Hulk (7'0", 1040 pounds, 104 BMI )
Bradley Wiggins (6'3", 150 pounds, 19 BMI)

***[Student Handout]***

## Specific Objectives

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Students will understand that

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Students will be able to

- Use a multivariate mathematical model to estimate a value.

### Problem Situation: Weighing the Models

In this module, we will investigate a number of mathematical models. Some build on the linear and exponential models we saw in Module 7. Some are different types of models. We begin by investigating mathematical models that estimate body fat.

According to the RAND Corporation Study that you read about in the PNL, the proportion of Americans who are severely obese continues to increase. The article presented in Science Daily indicates that the Body Mass Index was used to classify individuals as obese. We will consider the formula for BMI as well as its relationship to other health measurements.

(1) The study focuses on weight and obesity issues. What measurement is being used to identify obesity? What is this measure trying to estimate? Why was an estimate used?

*Answer:*

*The idea of body fat percentage should arises in the discussion, if not, you will need to introduce it. According to Wikipedia, "The body fat percentage of a person or animal is the total weight of fat divided by total weight; body fat includes essential body fat and storage body fat."[1] Body fat percentage is really what should be measured because the BMI and other approximations don't account for muscle mass, gender, and other important variables that might be useful in determining of someone is obese. However, measuring body fat percentage is difficult to do. You can look at the long description of measurements on the same Wikipedia page if you are interested. Most methods to estimate body fat involve a trade-off between convenience and accuracy. BMI was used in the study because it was convenient and is related with body fat percentage.*

***[Student Handout]***

1 http://en.wikipedia.org/wiki/Body_fat_percentage

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(2) What variables are used in the measurement of the BMI? How were these variables obtained for the people in the RAND study? What other variables might be important in determining if someone is obese or not?

*Answer:*

*BMI =weight/height$^2$, where weight is measured in kilograms and height is measured in meters. Body fat percentage is probably also based on other measurements (such as, difference in density, difference in electrical impedance, thickness of skin folds). Age, gender, ethnicity may also influence whether someone is obese. Answers will vary on how the data was obtained. Try to get students to imagine themselves as the researchers. They want a large, varied sample, and have limited funds. What would be a good way to get people's height and weight from a diversity of U.S. locations? From the actual study it says that: "The RAND study is based on the Behavioral Risk Factor Surveillance Survey (BRFSS), an annual survey conducted by the U.S. Centers for Disease Control and Prevention. The BRFSS, the world's largest annual telephone survey, tracks health risks in the United States. Height and weight is based on self-reporting. More than 3 million respondents were included in the analysis for the last decade. " (See http://www.rand.org/news/press/2012/10/01/index1.html.) You might point out the possible bias and error in self-reporting, but again not the trade off in getting data quickly. Researchers might also see reluctance to getting a large sample of people to come in and get weighed, for example. Subjects would likely tell you their height/weight over the phone for free, but you would need to pay them to come in so that you could take the measurements.*

***[Student Handout]***

(3) Compare the BMIs of the athletes whose heights and weights you brought to class. What do you notice?

*Student answers will vary, but most of the results should show BMI values in the 20s. Note that using the unit conversion approximations in the PNL (between inches and meters and pounds and kilograms) may result in about a 3% relative error for BMI calculations, but that will be close enough for the purposes here. Students could work in groups or individually and the results collected together in the following discussion. Students with access to laptops may wish to use Excel to do the computation, or you may ask students to enter their answers on a common spreadsheet that the whole class can view.*

***[Student Handout]***

(4) The Center for Disease Control (commonly referred to as the CDC) suggests that a BMI below 18.5 is Underweight; a BMI between 18.5 and 24.9 is Normal; a BMI between 25 and 29.9 is Overweight; and a BMI of 30 or above is Obese. What are these athlete's classifications based on their BMIs.

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*Again, student answers will vary. The classifications for the athletes listed in the Lesson Outline can be found in the table in answer to Question 6.*

***[Student Handout]***

(5) Using BMI to classify whether a person is obese is based on a relationship between BMI and body fat percentage. There are formulas that try to account for additional variables when estimating body fat. One such formula is the one presented by Deurenberg and Weststrate[2]. Use this formula to estimate your athlete's body fat percentages. If you are unsure of your athlete's age, make an estimate.

Men's Adult Body Fat % = 1.2 BMI + 0.23 Age – 16.2.
Women's Adult Body Fat % = 1.2 BMI + 0.23 Age – 5.4.

*As before, answers will vary. The body fat percentages for athletes in the Lesson Outline can be found in the table in answer to Question 6. We used 30 as an estimated age for these athletes.*

***[Student Handout]***

(6) The American Council on Exercise[3] classifies Body Fat Percentages as follows:

| **Description** | **Women** | **Men** |
|---|---|---|
| Essential Fat | 10–13% | 2–5% |
| Athletes | 14–20% | 6–13% |
| Fitness | 21–24% | 14–17% |
| Average | 25–31% | 18–24% |
| Obese | 32%+ | 25%+ |

What is your athlete's classification based on the estimated body fat percentage?

*Answers will vary depending on students' data. Below is the list for some athletes:*

| ***Name*** | ***BMI*** | ***BMI Class*** | ***Estimated Body Fat %*** | ***BF% Class*** |
|---|---|---|---|---|
| *Favre* | *27* | *Overweight* | *23* | *Average* |
| *Sorenstam* | *20* | *Normal* | *26* | *Average* |

2 http://www.halls.md/bmi/fat.htm

3 http://www.acefitness.org/blog/112/what-are-the-guidelines-for-percentage-of-body-fat/

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| | | | | |
|---|---|---|---|---|
| *Jordan* | *23* | *Normal* | *18* | *Average* |
| *Joyner-Kersee* | *22* | *Normal* | *28* | *Average* |
| *Ali* | *26* | *Overweight* | *22* | *Average* |
| *Navratilova* | *22* | *Normal* | *28* | *Average* |
| *Woods* | *24* | *Normal* | *20* | *Average* |
| *Hamm* | *21* | *Normal* | *27* | *Average* |
| *Hogan* | *34* | *Obese* | *32* | *Obese* |
| *Hulk* | *104* | *Obese* | *116* | *Obese* |
| *Wiggins* | *21* | *Normal* | *16* | *Fitness* |

***[Student Handout]***

(7) For the athletes, does their classification based on their BMI match their classification based on their estimated body fat percentage?

*Answer: In most cases, the classifications are the same (Normal matching Average). The Percentage Body Fat classification appears to have a broader "Average" range than the BMI "Normal" range. You should point out any outliers. In the above dataset, Hulk stands out because it would be impossible to have over 100% body fat.*

***[Student Handout]***

(8) Recall the cyclist Bradley Wiggins. In the PNL you calculated his BMI to be about 21.

(a) What is estimated body fat percentage, using Deurenberg and Weststrate's formula? Use the age of 32.

(b) One online article reported that Wiggins has a body fat percentage of 4%.[4] How does this compare to the estimations you made? What could explain any possible differences?

*Answers:*

*(a) 1.2*21+0.23*32-16.2=16.36 or about 16% body fat.*

*(b) It's hard to say why the numbers are so far off. Some may be the difference in age and weight of Wiggins. It most likely is due to very different ways of approximating body fat percentage. It's likely Wiggins had access to more expensive measurement techniques than using an estimate based on just his height, weight, and age. He was at a point where weight slowed him down and speed was essential. So, it's something he pays particular attention to: how lean he can get without loosing muscle.*

***[Student Handout]***

4 http://www.guardian.co.uk/sport/blog/2009/jul/19/bradley-wiggins-tour-de-france

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(9) Do you think the BMI a reasonable estimate of body fat percentage? Does it really help classify people as "overweight"? What good does it do? What harm does it do?

*Answer: For most people, BMI is a reasonable estimate. However, for people in the "extreme" categories it may not be as accurate. This is an important point because the PNL article was about the rise in those in the "extreme" obesity category. The advantage of the BMI is that it is easy to calculate, anyone can use the measure to gauge their weight classification. But, because it is not 100% accurate, people may have an inaccurate portrayal of their weight classification.*

Point out the formulas used for the BMI and estimating Body Fat Percentage are mathematical models. It should be emphasized that when students are substituting numbers into the formulas, that they are making use of a mathematical model that has been shown to represent fairly closely the real situation.

***[Student Handout]***

(10) Consider the verbal description of the BMI calculation given in the PNL reading: "weight in kilograms divided by the square of height in meters." Let's look a bit more at this mathematical model.

(a) Write the verbal description in the form of an equation.

(b) Consider the following scenarios, which person has the lower BMI in each case?

(i) Julie and Sarah have the same weight, but Julie is taller.
(ii) Julie and Sarah have the same height, but Julie weighs more.

(c) If you wanted to lower your BMI, which variable could you reasonably change? That is, which variable can vary for a fixed person?

(d) There are lots of ways where the calculation of the BMI can introduce errors simply by how far one rounds. Recalculate the BMI for your athlete by rounding every number at every step a bit more than you normally do. (For example, if he/she were 134 pounds, use 130 pounds instead. Use 1 inch as approximately 0.03 meters instead of 0.0254 meters, etc.) How does your answer, using rounded numbers, differ from the answer you first got during the PNL? Is this something you should worry about when doing this calculation?

*Answers: (a) The equation is* $BMI = \frac{w}{h^2}$ *. Let the students use their own variables and encourage them to choose letters that are meaningful the scenario. (b) (i) Julie; (ii) Sarah. (c) Height is pretty much fixed for most adults. So, the weight is the only real variable for a given person. (d) There are no right or wrong answers here and students should be encouraged to play around and see when and if rounding matters. Notably, approximating the height is likely to see the greatest difference because of the nature of squaring the number.*

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***[Student Handout]***

**Making Connections**

Record the important mathematical ideas from the discussion.

## Making Connections: Main Ideas to Highlight

**Body Mass Index and Body Fat Percentage are two ways to measure a person's size and weight. They take into account multiple variables, such as height, age, or gender, to classify people in different weight ranges. The values can be used as measurements of health.**

**Mathematical models are often not perfectly accurate ways to measure values, but they are useful and many have been shown to be calculate values close to the real data.**

Facilitation Prompts

- What are some of the benefits of using models such as the BMI and Body Fat Percentage to measure a person's size?
- Why might such formulas be convenient? Are there indicators of a person's health with regards to weight that the two models don't take into account?

When using a mathematical model, you need to be aware of when trade offs are being made between convenience and accuracy and judge whether those trade offs are reasonable. While BMI is not a perfect measurement of obesity, it is easier to determine than body fat percentage and for many individuals, there is a clear relationship between the two values. However, in cases where the individual has an extremely high or low body fat percentage, the use of BMI as a measure of general fitness can be misleading.

Often using multiple sources of information (height and weight in this example) can be used to make up for lacking a single measurement (body fat percentage in this example). This type of modeling is used frequently in health sciences where direct measurement of a physical feature would be difficult or impossible.

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## Key to OCE

(1) (a) The percentage will increase over time; (b) Yamana, the sister; (c) BMI has a greater impact..

(2) (a) A = sqrt((48*75)/3131) ≈ 1.07 $m^2$; (b) D= 25*1.07/1.73=15.46mg; (c) A = sqrt((4*75)/3131) ≈ 0.31 $m^2$; (d) D = 25*1.07/1.73 = 4.47mg (e) The child got less than 30% of the dosage that he/she needed. So, the prescription probably wasn't effective. Note, the result could have been a lot worse if the error was in the other direction.

## Key to PNL

(3) Answers will vary.

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