Lesson 3.6: Balancing Blood Alcohol
Theme: Medical Literacy

Main Math Topic	Main Quantitative Reasoning Context	Productive Persistence Focus	Level of Productive Struggle
Solving linear equations	Blood alcohol content	Not applicable	Level 3

Prerequisite Assumptions

Before beginning this lesson, students should

- understand the use of variables in mathematical equations.
- be able to substitute a value for a variable in a mathematical equation and simplify the equation.
- understand that an equation is a statement of equality.

Specific Objectives

Students will understand that

- addition/subtraction and multiplication/division are inverse operations.
- solving for a variable includes isolating it by "undoing" the actions to it.

Students will be able to

- solve for a variable in a linear equation.
- explicitly write out order of operations to evaluate a given equation.

Explicit Connections

- Solving equations is based on the principles of undoing operations (and steps) and on balancing each operation.

Notes to Self

One thing I want to do during this lesson …

One thing I want to pay attention to in my students' thinking …

One connection or idea I want to remember …

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Suggested Timeline

Duration	Activity	Suggested Structure
5 minutes	Problem Situation and Question 1	Class discussion
20 minutes	Work on and discuss Questions 2–4	Small groups, class discussion
10 minutes	Work on and discuss Question 5	Small groups, class discussion, demonstration
10 minutes	Practice by varying Question 5 and exploring the scenario in Question 6	Small groups
5 minutes	Making Connections	Class discussion

[Student Handout]

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Problem Situation: Calculating Blood Alcohol Content

Blood alcohol content (BAC) is a measurement of how much alcohol is in someone's blood. It is usually measured as a percentage. So, a BAC of 0.3% is three-tenths of 1%. That is, there are 3 grams of alcohol for every 1,000 grams of blood. A BAC of 0.05% impairs reasoning and the ability to concentrate. A BAC of 0.30% can lead to a blackout, shortness of breath, and loss of bladder control. In most states, the legal limit for driving is a BAC of 0.08%.[1]

BAC is usually determined by a breathalyzer, urinalysis, or blood test. However, Swedish physician, E.M.P. Widmark developed the following equation for *estimating* an individual's BAC. This formula is widely used by forensic scientists:[2]

$$B = 0.015t + \left(\frac{2.84N}{Wg}\right)$$

where

B = percentage of BAC

N = number of "standard drinks" (A standard drink is one 12-ounce beer, one 5-ounce glass of wine, or one 1.5-ounce shot of liquor.) *N* should be at least 1.

[1] Retrieved from http://en.wikipedia.org/wiki/Blood_alcohol_content.

[2] Retrieved from www.icadts2007.org/print/108widmarksequation.pdf

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W = weight in pounds

g = gender constant, 0.68 for men and 0.55 for women

t = number of hours since the first drink

(1) Looking at the equation, discuss why the items on the right of the equation make sense in calculating BAC.

Carefully discuss each variable, determining why it might be important to the BAC calculation. The variables *B*, *N*, *W*, *g*, and *t* change depending on the person. Ensure that students understand that the units of *B* are a percentage. So, 0.08 is not 8%, but eight-hundredths of a percent.

Notes on the Widmark Equation:

- This equation has been simplified from the one found in the reference.
- The numbers 0.015 and 2.84 are constants based on the average person. The value 0.015 is the average rate of elimination of alcohol, and the value 2.84 is a conversion factor between weight (in pounds) of an individual, density of alcohol in a standard drink, and density of water in an average person.
- Many high-end beers have twice as much alcohol as the "standard beer," which assumes 4% alcohol.

[Student Handout]

(2) Consider the case of a male student who has three beers and weighs 120 pounds. Simplify the equation as much as possible for this case. What variables are still unknown in the equation?

This problem should be a review of earlier modules. The simplified equation is

$$B = -0.015 \cdot t + 0.104$$

Be sure, though, that everyone has the same simplified equation before moving on. In addition, point out that, in this case, you have held some variables (*N*, *W*, and *g*) fixed as a way of further investigating the model. Now, there are only two unknowns in the equation.

[Student Handout]

(3) Using your simplified equation, find the estimated BAC for this student one, three, and five hours after his first drink. What patterns do you notice in the data?

Answers: When t = 1, BAC = 0.089; when t = 3, BAC = 0.059; and when t = 5, BAC = 0.029.

Notice that 0.089 is above the legal limit for driving and 0.059 is still high (0.05 is the legal limit in Canada). Point out that they "solved" for *B* when they substituted in *t*. That is, *B* was the only unknown left in the equation. Students should notice that BAC decreases over time. Students might try

substituting in more hours. If they try any time greater than seven hours ($t \geq 7$), the BAC is negative. Talk about how to interpret this value. After a certain point, the BAC is 0 because the alcohol has been metabolized by the liver, even if the equation gives a negative number for BAC.

[Student Handout]

(4) Discuss with your group how you arrived at the BAC values mathematically. For example, did you multiply, add, subtract, etc., and what did you do first? Outline the steps that you took to get from the time to the BAC.

Answer

Students should outline something like this:

- *First, replace* t *with the given hours in the equation (i.e., plug in the value for* t*).*
- *Second, multiply the hours by −0.015.*
- *Third, add 0.104.*
- *The result is the BAC.*

This may seem tedious, but it helps students when they try to figure out how to solve for *t* when *B* is known. Ensure that students are being explicit about the order of operations and remind them how to determine this from the equation. It is important that they see the relationship between the equation and the steps. Also, reinforce that the steps are based on the order of operations.

[Student Handout]

(5) How long will it take for this student's BAC to be 0.08, the legal limit? How long will it take for the alcohol to be completely metabolized resulting in a BAC of 0.0?

Answer: t = 1.6 hours for BAC = 0.08 and t = 6.93 hours for BAC = 0.0.

Let students struggle with this question a bit. Some will plug in hours until they come close to 0.08 and 0.0, but they will not get those BAC values exactly. Direct them back to the equation, asking them what is unknown in this case. Ask them to look at their algorithm above and discuss how finding the hours when given the BAC is really working backwards through their algorithm and undoing each step. To get students started, you may need to ask what undoes addition and what undoes multiplication. Introduce the term *inverse operation* in talking about undoing. You can ask them to write out the undoing steps such as:

- First, start with the known value of BAC for *B* in equation.
- Second, subtract 0.104 (the other way, they had added 0.124).
- Third, divide by −0.015 (because division undoes multiplication).
- The result is *t*.

Although some students may have solved similar equations algebraically in the past, you should

explicitly and carefully model how this undoing works, emphasizing the importance of keeping both sides of the equation balanced using the *B* = 0.08 case. Then, give them a chance to work through the case of *B* = 0 on their own before moving to the next question. A demonstration on the board that links their algorithm with the algebraic operations might look like this:

Undoing Algorithm	Looks Like with the Equation	Notes
Step 1: Replace given BAC for *B*	$0.08 = -0.015 \cdot t + 0.104$	Substituting *B* into the equation.
Step 2: Subtract 0.104	$0.08 - 0.104 = -0.015 \cdot t + 0.104 - 0.1$ and simplified: $-0.024 = -0.015 \cdot t$	To maintain equality, any operation you do on one side must also be done to the other side of the equation.
Step 3: Divide by −0.015	$\frac{-0.024}{-0.015} = \frac{-0.015t}{-0.015}$ or $1.6 = t$	Again, dividing both sides of the equation by −0.015 to maintain a balanced and equal equation.
The result is *t*	*t* = 1.6 hours	By convention, you rewrite so that the variable is on the left of the equation.

Undoing (inverse operations) and balancing equations are two key ideas that students can use to solve for an unknown variable nested in an equation. It is also important for students to realize that undoing is directly related to the order of operations—in essence, performing the order of operations backwards. One strategy in deciding what to do first in solving an equation is to first put a value in for the variable and simplify as modeled in this lesson. The steps in simplifying (the order of operations) are reversed for solving.

Make sure to remind students that 1.6 hours is *not* 1 hour and 60 minutes. If there is time, you could ask them to convert 1.6 hours into 1 hour and 36 minutes. Ask students to check the reasonableness of their answers with the numbers they got in the "going forward" model above.

Ample time should be spent on Question 5. For students ready to move on, direct them to Question 6. For students who need more practice, stick with the above model and provide them varying values for *B*.

[Student Handout]

(6) A female student, weighing 110 pounds, plans on going home in two hours. Using the formula above, the simplified equation for this case is

$$B = -0.03 + \frac{2.84 \cdot N}{60.5}$$

(a) Compare her BAC for one glass of wine versus three glasses of wine at the time she will leave.

Answer: When N = 1, B = 0.017; when N = 3, B = 0.111.

Point out how, in this case, you have held *t* fixed and let *N* be the variable. Students should notice that if they fix *N* and let time increase, BAC goes down (as seen above). If they fix *t* and let *N* increase, BAC goes up (as seen in this case). Talk about why this makes sense. Finally, if needed, remind students that "adding a negative 0.03" is equivalent to "subtracting positive 0.03." This might help them in the undoing process in Part (b).

[Student Handout]

(b) In this scenario, determine how many drinks she can have so that her BAC remains less than 0.08.

When B = 0.08, N = 2.34 drinks. Ask students to estimate what 2.34 drinks means.

Solving for *N* in this equation is a bit trickier than before because the unknown is in the second half of the equation and has been multiplied and divided by different numbers. If students get stuck, remind them about order of operations and encourage them to write out the steps going from *N* to *B* and then back from *B* to *N*. Some students may recognize that they can divide the coefficient of *N* to get rid of the fraction. This is fine, but it is important that you discuss how to solve the equation by undoing the division. They will use this when solving proportions in Lesson 3.7.

[Student Handout]

Making Connections

Record the important mathematical ideas from the discussion.

Making Connections: Main Ideas to Highlight

Solving equations is based on the principles of undoing operations (and steps) and balancing each operation.

In Lesson 1.4, students were introduced to the idea of creating equivalent expressions based on mathematical rules. This lesson builds on that idea. Each step of solving an equation creates a new equivalent form of the original equation. Students must understand that the rules that govern solving equations apply to all equations. This will be emphasized over the next two lessons.

Facilitation Prompts

- In an equation with two operations (use an example from the lesson), how do you decide which operation to do first? It can be useful to demonstrate that the result is different if the work is done in a different order.
- How do you decide which operation to use? What if negatives are involved?
- Why do you have to perform the same operation on both sides of the equation?
- How can you check if a solution to an equation is correct?

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[Student Handout]

Further Applications

(1) Solve the following equation for the values given in Parts (a) and (b). In each case, write the steps you used as you did in Question 4 from the lesson.

$$y = -4x - 2$$

(a) Solve for y if $x = -3$. Write your steps.

(b) Solve for x if $y = -3$. Write your steps.

Answers: (a) Substitute −3 in for x. Multiply −4 times −3. Subtract 2. y = 10

(b) Substitute −3 in for y. *Add 2 to both sides of the equation. Divide both sides by −4.* x *= 1/4*

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Key to OCE

(1) iv

(2) Answers will vary.

(3) (a) 3; (b) 5/2; (c) −3; (d) 20

(4) (a) 1 gram of alcohol for every 1,000 grams of blood; (b) 0.2 grams of alcohol for every 1,000 grams of blood

(5) excerpts from the website are shown below

Progressive effects of alcohol[1]		
BAC (% by vol.)	**Behavior**	**Impairment**
(c) BAC = 0.05 0.030–0.059	Mild euphoria Relaxation Joyousness Talkativeness Decreased inhibition	Concentration
(a) BAC = 0.1 0.10–0.19	Over-expression Emotional swings Anger or sadness Boisterousness Decreased libido	Reflexes Reaction time Gross motor control Staggering Slurred speech
(b) BAC = 0.5 0.40–0.50	General lack of behavior Unconsciousness Death is possible	Breathing Heart rate

(6) i (3.33–3.37 hours depending on rounding)

(7) (a) Answers will vary. (b) Answers will vary.

(8)

Year	% of Retired People
1965	10%
2009	15%
2052	20%

(9) (a) Perimeter is 2(14 + 9/12) + 2(13) = 55.5 ft; (b) 10% waste allowance is 1.10(55.5) = 61.05 ft. 8 boards is 64 ft, so 8 boards needed; (c) Cost is 8(24)(1.0825) = $207.84.

(10) (a) Answers will vary. **Example:** $P = 2L + 2W$ with P = perimeter, L = length, W = width; (b) 90 ft

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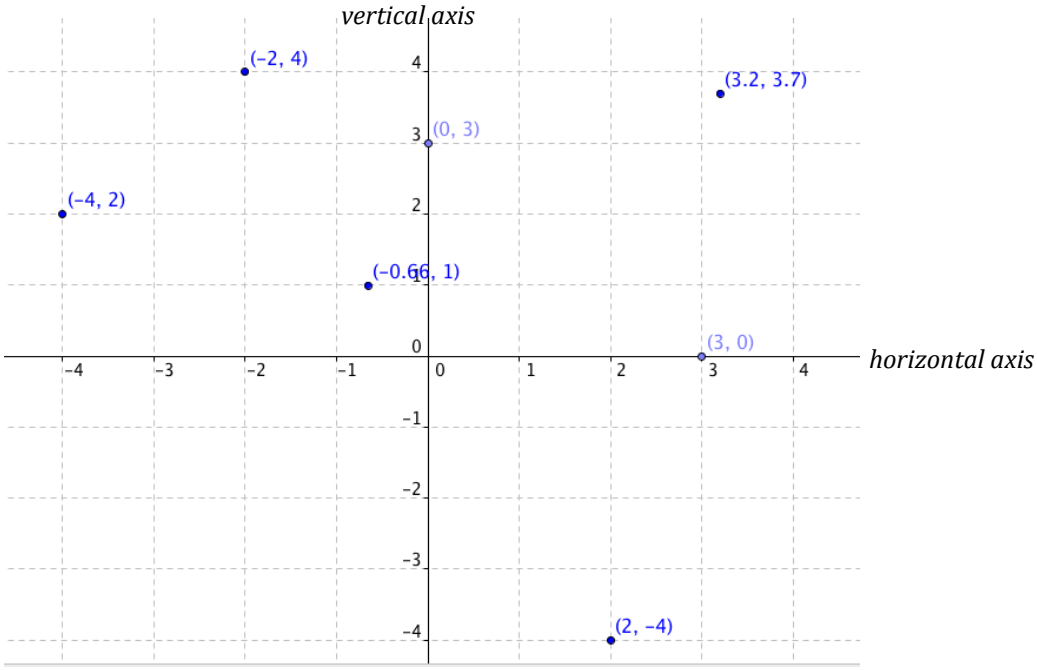
(11)



(12) (a) 35x + 75 or 75 + 35x; (b) The variable represents the number of weeks that Ben makes a deposit. (c) ii, iii; (d) 19 weeks

(13) (a) 5:9 or $\frac{5}{9}$; (b) Answers will vary.

(14) i, iv, v

(15) Answers will vary.

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