Lesson 11.1.1

Introduction to Chi-Square Tests for One-Way Tables

STUDENT NAME ______________________ **DATE** ______________

INTRODUCTION

Manufacturing Cereal

In a previous lesson, you were asked to design a children's cereal that struck an effective balance of taste and nutrition in order to receive a favorable rating from *Consumer Reports*. In this lesson, you will discuss some aspects of effectively marketing the cereal to consumers. Effective marketing is important because the goal is to keep costs low while increasing sales and profits.

It costs too much to make many different kinds of shapes for cereal. Because of manufacturing costs and concerns, all the cereal pieces, in the cereal that you develop, will be the same in terms of size, shape, and weight. However, suppose that the research shows that certain colors and certain distributions of colors are more appealing to consumers and are relatively inexpensive to produce.

Assume that the desired color distribution for the cereal is **20% red, 35% white,** and **45% blue.** To check these proportions, you take a simple random sample of 1,000 cereal pieces and count how many of each color are present in the sample. When there is strong evidence *against* the claim that the population proportions are 20% red, 35% white, and 45% blue this implies that something may be wrong with the production process.

TRY THESE

1 How many pieces of each color would you expect in your sample of 1,000 pieces if the true population proportions were in fact 20% red, 35% white, and 45% blue? (**Note:** These three expected counts should add up to 1,000 since you have 1,000 pieces in your sample.)

2 If you obtained a sample of 1,000 pieces that exactly matched the three counts you computed in Question 1, it would *not* suggest that there are any problems in the manufacturing process since your actual counts were the same as your expected counts.

Recall that you have discussed sampling variability in a previous module. You know that it is unusual to obtain a sample with the exact counts you just computed in Question 1.

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A For a sample of 1,000 pieces, give another example of counts for red, white, and blue pieces that *would not* suggest that there are any problems in the manufacturing process.

Red	White	Blue

B For a sample of 1000 pieces, give an *example* of counts for red, white, and blue pieces that would suggest that there are some problems in the manufacturing process.

Red	White	Blue

3 The tables that follow show the distributions of four random samples. Each sample has a size of 1,000 and is taken from four different production days.

Do the following for each sample:

- First, examine the distribution of the actual counts obtained from the sample in the table.
- Decide whether you think the sample provides strong evidence, moderate evidence, or weak evidence *against* the claim that the population proportions are 20% red, 35% white, and 45% blue and record your response in the space provided.
- Finally, list the characteristics of the sample that led you to your decision.

Sample: Day 1

Red	White	Blue
210	360	430

It appears that the Day 1 sample provides *(circle one)*

strong **moderate** **weak**

evidence *against* the claim that the population proportions are 20% red, 35% white, and 45% blue because …

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Sample: Day 2

Red	White	Blue
200	425	375

It appears that the Day 2 sample provides *(circle one)*

strong **moderate** **weak**

evidence *against* the claim that the population proportions are 20% red, 35% white, and 45% blue because …

Sample: Day 3

Red	White	Blue
180	360	460

It appears that the Day 3 sample provides *(circle one)*

strong **moderate** **weak**

evidence *against* the claim that the population proportions are 20% red, 35% white, and 45% blue because …

Sample: Day 4

Red	White	Blue
235	355	410

It appears that the Day 4 sample provides *(circle one)*

strong **moderate** **weak**

evidence *against* the claim that the population proportions are 20% red, 35% white, and 45% blue because …

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4 Based on your work in Question 3, which sample gives the strongest evidence *against* the claim that the population proportions are 20% red, 35% white, and 45% blue? What characteristics of the sample were most important in your decision?

5 Sometimes, in the production process, you may not get the distribution of 20% red, 35% white, and 45% blue that you had hoped to achieve. When might you become concerned about a large difference? Would you be concerned if an actual sample count differed from its expected count by about 10 pieces? How about by about 50 pieces or 100 pieces? Why? Explain your reasoning.

6 It would be useful to have a statistical measure of deviation. The statistical measure of deviation helps you decide how much the distribution of a sample (such as those shown previously) deviates from what is expected.
Create a method (and/or a statistic) to *measure* which sample deviates the most from the ideal expected distribution.

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Quantifying the Strength of the Evidence

Earlier in this lesson, we talked about strong evidence against a claim about population proportions. Strong evidence against the claim that the population proportions are 20% red, 35% white, and 45% blue implies that something may be wrong with the production process.

For each sample, you can compute a chi-square statistic (*chi* is pronounced *ki* as in *kite*). The chi-square statistic is useful in assessing the strength of your evidence that something might be wrong with the production process. The chi-square statistic relies on the comparison of *how different an observed count in a given category is from the count that was expected for that given category*.
Keep this information in mind. In later lessons, you will use this statistic in a way that is similar to how you used *Z*-scores for testing hypotheses in previous modules.

Important Information: The steps for computing a chi-square value (written as χ^2) are as follows:

(1) For each category, compute the difference between the observed, or actual, count for that category (obtained from the sample) and the expected count for that category:

$$\textit{Observed Count} - \textit{Expected Count}$$

(2) For each category, compute the square of this difference obtained in Step 1:

$$(\textit{Observed Count} - \textit{Expected Count})^2$$

(3) For each category, divide the squared difference obtained in Step 2 by the expected count for the category:

$$(\textit{Observed Count} - \textit{Expected Count})^2/\textit{Expected Count}$$

(4) Add up the Step 3 calculation results from each category; this is the chi-square value.

Example

Using the Day 1 sample data set as an example:

	Red	White	Blue
Observed Count (from sample)	210	360	430
Expected Count (based on desired population distribution)	200	350	450
Step 1: Observed Count - Expected Count	10	10	-20
Step 2: (Observed Count - Expected Count)^2	100	100	400
Step 3: (Observed Count - Expected Count)^2/Expected Count	0.5	0.286	0.889

Step 4: 0.5 + 0.286 + 0.889 = 1.675

For the Day 1 sample, the chi-square value generated is 1.675.

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Use the steps above to answer the following questions:

7 Compute the chi-square values for the other samples (Days 2–4) by filling in the tables below. Compute the Step 3 calculations to three decimal places as shown in the previous example.

Day 2

	Red	White	Blue
Observed Count (from sample)	200	425	375
Expected Count (based on desired population distribution)	200	350	450
Step 1: Observed Count – Expected Count	0	75	-75
Step 2: (Observed Count –Expected Count)^2	0	5625	5625
Step 3: (Observed Count – Expected Count)^2/Expected Count	0		

Step 4: 0 + + = 28.571

For the Day 2 sample, the chi-square value generated is 28.571.

Day 3

	Red	White	Blue
Observed Count (from sample)	180	360	460
Expected Count (based on desired population distribution)	200	350	450
Step 1: Observed Count - Expected Count			10
Step 2: (Observed Count - Expected Count)^2	400		100
Step 3: (Observed Count - Expected Count)^2/Expected Count			

Step 4: + + =

For the Day 3 sample, the chi-square value generated is _____________.

Day 4

	Red	White	Blue
Observed Count (from sample)	235	355	410
Expected Count (based on desired population distribution)	200	350	450
Step 1: Observed Count – Expected Count			
Step 2: (Observed Count –Expected Count)^2			
Step 3: (Observed Count – Expected Count)^2/Expected Count			

Step 4: + + = 9.752

For the Day 4 sample, the chi-square value generated is 9.752.

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8 Which of the daily samples had the highest chi-square value?

First, think about the size of the chi-square values for these four samples. Do they appear to correspond in any way to the initial strength-of-evidence statements that you made in Question 3 about the cereal population proportions?

9 What happens if one category has an observed count and an expected count that are exactly the same? Is it possible to still have a large chi-square value generated by that sample? If so, make up a sample that shows an observed count and an expected count that are exactly the same. Did any of the samples in Question 7 exhibit this behavior?

10 Now suppose that no category in a sample has a case where the observed count and expected count are exactly the same. Is it still possible to have a small chi-square value generated by that sample? If so, make up a sample that shows how this could happen. Did any of the samples in Question 7 exhibit this behavior?

11 In the Day 2 data set, the white and blue category counts had an actual count that differed from the expected count by 75 pieces.

Which difference made a larger contribution to the size of the chi-square value:
- the difference of size 75 relative to an expected count of 350 (white)
- or the difference of size 75 relative to an expected count of 450 (blue)?

In other words, would a difference of 75 make a larger contribution to the size of the chi-square value for a category with an expected count of 350 than it would be for a category with an expected count of 450? Explain your thinking.

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TAKE IT HOME

1 Based on the samples you worked on today, answer the following questions:

A Which seems to provide greater evidence *against* the claim that the population proportions are 20% red, 35% white, and 45% blue:
- a high chi-square value (such as you computed for Days 2 and 4)
- or a low chi-square value (such as you computed for Days 1 and 3)?

B Earlier in this lesson we learned that strong evidence against the claim that the population proportions are 20% red, 35% white, and 45% blue implies that something may be wrong with the production process.
Based on your answer above, would a high chi-square value or a low chi-square value give stronger evidence that something may be wrong with the production process?

C Is it possible to obtain a *negative* chi-square statistic? If so, explain how. If not, explain why not.

2 On Day 5, the following sample was collected:

Red	White	Blue
178	364	458

Compute the chi-square value. Determine if this value provides strong evidence against the claim that the population proportions for the cereal pieces are 20% red, 35% white, and 45% blue.

(**Note**: For reasons that will be explained in a future lesson, in a case where the categorical variable contains three categories:
- consider a chi-square value of 5.99 or greater to be statistically significant evidence against the claim that the population proportions of cereal pieces are 20% red, 35% white, and 45% blue.)

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